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## **Development of analysis tools for active shape and vibration control**

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# DEVELOPMENT OF ANALYSIS TOOLS FOR ACTIVE SHAPE AND VIBRATION CONTROL

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## Abstract

Active shape and vibration control are means for obtaining optimal flow conditions around wings, ducts and channels under different conditions. This means that the structure can be adapted (deformed or damped) such that aerodynamic or vibro-acoustic behaviour is optimal for that particular situation. The fast developments in computer technology makes it possible that more complex analyses aerodynamics and vibro-acoustics included are applied in the design process. At NLR research is carried out on the integration of advanced analysis tools in design environments. In this paper the tools which are developed for the analysis of active shape and vibration control are presented.

The backbone of the design environment is an optimisation algorithm which helps the designer to come up with optimal designs of structures. In the case of active shape and vibration control the optimal design of controllers is a new aspect. This means that in addition to the optimisation of the locations of sensors and actuators the control parameters have to be optimised. In this paper a method is proposed to optimise locations and control parameters at once with the standard finite element representation of the equations of motion as a base.

## 1. Introduction

Active shape and vibration control are means for obtaining optimal flow conditions around wings, ducts and channels under different conditions. This means that the structure can be adapted (deformed or damped) such that aerodynamic or vibro-acoustic behaviour is optimal for that particular situation. It is no common practice to analyse the effect of active shape and vibration control in the design process due to the complexity of the algorithms and the high computation times. However, the fast developments in computer technology makes it possible that more complex analyses can be applied in the design process.

At NLR research is carried out on the integration of advanced analysis tools in design environments. This started at the end of the eighties with a multi-level optimisation tool for preliminary design of aircraft structures (Ref. 1). Currently this tool is extended to multi-disciplinary analyses and optimisation (MDO) like aero-elasticity and vibro-acoustics (Ref. 2, 3).

Recently a study has been started to incorporate the analyses and optimisation of 'active structures' in the MDO environment. The basis for this study is the knowledge and

developed analysis tools obtained from NLR research on optimisation of damping treatments and piezoelectric materials. This will be presented in the first part of this paper. By implementing this knowledge in a MDO environment NLR wants to make it available for the engineer in the industry.

A new aspect which comes up with active shape adaptation and active vibration damping is control. In the second part of this paper attention is paid to the incorporation of control in the equation of motion and the optimisation analysis.

## 2. Finite element formulation of piezoelectric elements

To be able to simulate active damping, at NLR a finite element method has been developed with which piezoelectric material behaviour can be applied for all existing structural finite elements. This is realised by creating piezoelectric elements as so-called "overlay" elements.

The finite element representation of the equations of motion for a piezoelectric element can be obtained by the assembly of the structural part and the electrical part of the element matrices as depicted in equation (1).

$$\begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix} \begin{Bmatrix} u \\ \phi \end{Bmatrix} = \begin{Bmatrix} F \\ G \end{Bmatrix} \quad (1)$$

with: $[M_{uu}]$	the structural element mass matrix
$[K_{uu}]$	the structural element stiffness matrix
$[K_{\phi\phi}]$	the dielectric element matrix
$[K_{u\phi}]$	the piezoelectric element couple matrix
$\{u\}$	the mechanical nodal displacements
$\{\phi\}$	the electrical nodal potentials
$\{F\}$	the mechanical nodal load
$\{G\}$	the electrical nodal charge

The finite element representation of the equations of motion for a structure consisting of piezoelectric material can be obtained by the assembly of all the element matrices as depicted in equation (1) to global system matrices. The solution of this global set of equations yields the mechanical displacements and electrical potentials in the structure with piezoelectric material.

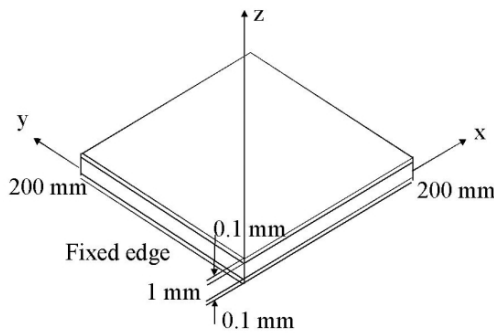
The two sets of linear equations from equation (1) are coupled by the piezoelectric element matrix  $[K_{u\phi}]$  and can be split into two separate matrix equations with diminishing piezoelectricity (piezoelectricity = 0  $\Rightarrow [K_{u\phi}] = 0$ ). These

two separate (independent) sets of equations describe respectively pure structural mechanics problems (first row of Eq. (1) with  $[K_{u\phi}] = 0$ ) and electrostatic field problems (second row of Eq. (1) with  $[K_{\phi u}] = 0$ ).

The matrices  $[M_{uu}]$  and  $[K_{uu}]$  are the standard structural element mass and stiffness matrices for a beam, shell or volume element which are available in finite element programs. This means that the element stiffness matrix of the piezoelectric element can be assembled from an existing element stiffness matrix and the general dielectric element and piezoelectric element couple matrices of the overlay element. In this way piezoelectric material behaviour can be simulated in combination with all existing structural beam, shell and volume elements and a general piezoelectric overlay element.

The overlay element has been implemented in the modular finite element program B2000. This is a modular finite element package which is used by the NLR as 'the computational mechanics laboratory', a development and test environment for computational mechanics.

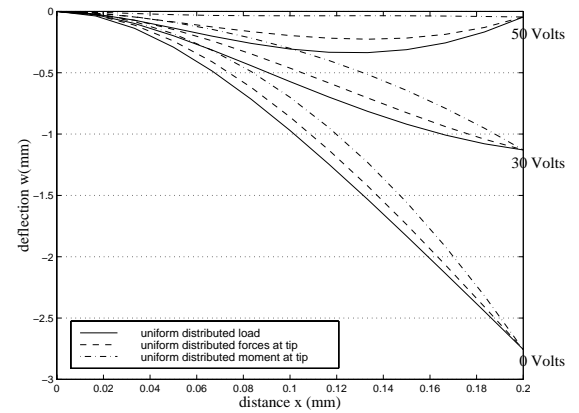
The piezoelectric overlay element has been tested and verified with test cases from literature. One of these test cases concerned the effect of piezoelectric material on the shape of a structure. It concerns a cantilevered laminated composite plate with on both the upper and lower surface a ceramic piezoelectric layer (see Fig. 1). Thirty six four node shell elements are used to model the plate. The composite plate is made of T200/976 graphite-epoxy composites with a stacking sequence of  $[-45/45-45/45]$ . Each layer has a thickness of 0.25 mm. The piezoelectric ceramic is PZT G1195N with a thickness of 0.1 mm.



**Figure 1** A laminated composite plate with piezoelectric layers

The plate is exposed to three different load cases namely a uniform distributed transverse load of  $100 \text{ N/m}^2$ , a tip force of  $0.1137 \text{ N}$  and a tip moment of  $0.01448 \text{ Nm}$ . The deflections of the plate are calculated when a voltage of 0 V, 30 V and 50 V is applied over the upper piezoelectric layer and an opposite voltage over the lower piezoelectric layer. Figure 2 shows the calculated centreline deflection of the composite plate under these different active input voltages. It is observed that for 50 V the tip deflection is almost zero. Further is shown that for 50 V combined with the tip moment the deflection of the whole centreline is almost zero. So, with a piezoelectric layer which covers the whole plate the deformation due to a constant voltage corresponds with the deformation caused by a tip moment, as expected from the equations.

**Figure 2** The centreline deflection under various loads

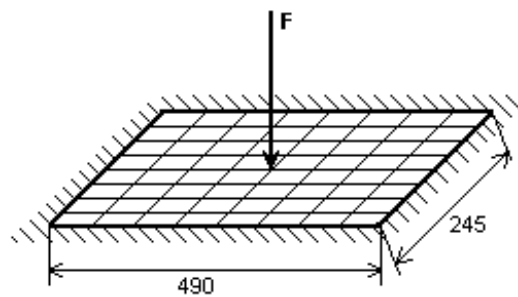


and actuator input voltages

### 3. Optimisation

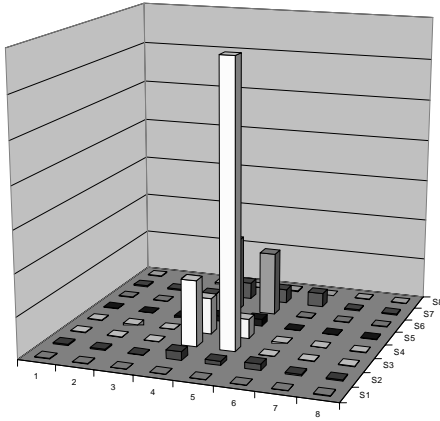
In the case piezoelectric material has to be added to a structure the mass of the structure will increase generally. Especially for light weight structures this is a disadvantage and therefore as less as possible piezoelectric material has to be added. This means that the actuator and/or sensor material to adapt the shape or vibration level has to be added at optimal locations.

At NLR research has been carried out in which the optimal locations of four tuned dampers on a vibrating rectangular plate have been calculated such that the response of the plate in a certain frequency range is minimal (Ref. 4). In principle this algorithm can be used for the determination of the optimal location of piezoelectric material, too. This has been evaluated in a pilot study on an on all sides clamped plate excited by a point force in the centre (see Fig. 3).



**Figure 3** Finite element mesh of an on four sides clamped plate excited by a point force in the centre

Initially the whole plate is covered with a piezoelectric material layer of constant thickness. The objective of the optimisation is to calculate the optimal thickness distribution of the piezoelectric material layer such that the response in the centre of the plate at the first eigenfrequency is 50 % of the initial response and the first eigenfrequency does not change. In figure 4 the final thickness distribution is depicted. As expected, this shows that the piezoelectric (actuator) material is concentrated around the location where the response has to be reduced. The asymmetric thickness distribution is probably caused by the coarse mesh and the strategy followed in the optimisation process.



**Figure 4** *Optimal thickness distribution for 50% reduction of the response in the plate centre (each patch corresponds with a finite element)*

In the case the response at another location of the plate has to be reduced the piezoelectric material is concentrated on that location.

This preliminary study has proven that the optimisation algorithms developed for the optimal positioning of tuned dampers can be used for the optimal placement of piezoelectric material, too. At NLR research is going on to apply other optimisation objectives such as reduction of the response of the whole plate in a certain frequency range or reduction of the radiated sound intensity or to obtain a certain shape of the geometry.

#### 4. Control

The basic ingredients for the analysis and design environment for active shape and vibration control are available in the analysis program B2000 namely static deformation, dynamic and vibro-acoustic analysis tools, optimisation algorithms, vibration reduction algorithms and piezoelectric overlay elements. The objective of this study is to combine these tools and to come up with a tool with which the effect of active shape and vibration control on the dynamic and acoustic behaviour of the structure can be predicted and optimised.

To perform a coupled structural dynamic and control optimisation in B2000 it is possible to incorporate the control parameters in the equations of motion. For a structure with actuators and sensors the finite element representation of these equations is:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} - \{F_c\} \quad (2)$$

with:  $[M]$  the mass matrix  
 $[C]$  the damping matrix  
 $[K]$  the stiffness matrix  
 $\{F\}$  the applied force  
 $\{F_c\}$  the applied force by the actuators  
 $\{x\}$  the vector with nodal displacements and voltages  
 • the derivative with respect to time

In the case of P(roportional)I(ntegrational)D(ifferential) control of the velocity, the actuator force can be written as:

$$\{F_c\} = [R_I]\{x\} + [R_P]\{\dot{x}\} + [R_D]\{\ddot{x}\} \quad (3)$$

with:  $[R_I]$  the matrix with parameters of the I(ntegrational) part of the control

$[R_P]$  the matrix with parameters of the P(roportional) part of the control

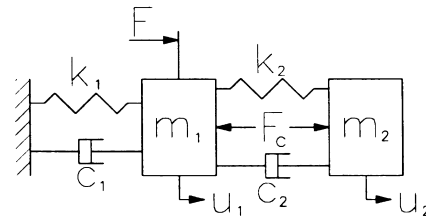
$[R_D]$  the matrix with parameters of the D(ifferential) part of the control

Substitution of eq. 3 into eq. 2 gives:

$$([M] + [R_D])\{\ddot{x}\} + ([C] + [R_P])\{\dot{x}\} + ([K] + [R_I])\{x\} = \{F\} \quad (4)$$

The values for the control parameters in the matrices  $[R_I]$ ,  $[R_P]$  and  $[R_D]$  can now be obtained with the same optimisation methods as used for the determination of the optimal locations of actuators and sensors.

In structural dynamics the dynamic behaviour of the structure is optimised by varying the structural properties such as thickness, mass density or damping ratio. This means that the components of the mass, damping and/or stiffness matrices are changed. With the same optimisation procedure the values for the control parameters in  $[R_I]$ ,  $[R_P]$  and  $[R_D]$  can be determined.



**Figure 5** *Two degrees of freedom mass, spring, dashpot system with control force.*

As an example the control parameters are determined for the two degrees of freedom (DOF) system as depicted in figure 5. The properties are summarised in table 1.

If full state feedback is applied with a PD controller, the control force  $F_c$  situated between mass 1 and mass 2 depends on the displacements and the velocities and can be written as:

$$F_c = \begin{bmatrix} r_1 & r_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + \begin{bmatrix} r_3 & r_4 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix}$$

The values of the control parameters can be determined with a special routine in MATLAB (the so-called pole placement routine). This is carried out for the case that the goal is to obtain a system with a certain value for the modal damping with the assumption that the undamped eigenfrequencies are equal to the damped ones. The results are depicted in the second row of table 2. The values of the control parameters obtained by optimisation of the standard equations of motion with the control parameters as unknowns in the stiffness and damping matrices are depicted in the second row of table 2. This table shows that for both methods approximately the same values for the control parameters are obtained.

At the moment research is going on to apply the optimisation approach in B2000 to determine values for the control parameters for more complex structures.



**Table 1** *Properties of the two degrees of freedom (DOF) system.*

$m_1$	= 2 kg	$m_2$	= 3 kg
$k_1$	= 10 N/m	$k_2$	= 15 N/m
$c_1$	= 0.5 Ns/m	$c_2$	= 0.1 Ns/m
$u_1(0)$	= 0 m	$u_2(0)$	= 0 m
$u'_1(0)$	= 0 m/s	$u'_2(0)$	= 0 m/s

**Table 2** *Calculated values for the control parameters obtained with pole placement method in MATLAB and optimisation of the equation of motions.*

Parameter	$r_1$	$r_2$	$r_3$	$r_4$
Values obtained with Pole placement	-0.66	0.00	0.10	5.44
Values obtained with optimisation	-0.62	-0.07	0.09	5.42

## 5. Conclusions

At NLR research is carried out on the integration of advanced analysis tools in design environments. The fast development in computer technology makes it possible to apply more complex analyses in such a design environment. Therefore a study has been started to incorporate in the design environment knowledge and analysis tools in the field of active shape and vibration control. In this way (theoretical) knowledge gathered by a research institute like the NLR comes available for the industry. The backbone of the design environment is an optimisation algorithm which helps the designer to come up with optimal designs of structures.

In the case of active shape and vibration control the optimal design of controllers is a new aspect. To incorporate this in the existing environment the control parameters have been included in the stiffness, damping and stiffness matrices of the finite element representation of the equations of motion. Preliminary results show that this approach is promising.

## 6. Acknowledgements

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